Algebra Round Problems 2026 IMT

1. Let a and b be the real roots of the quadratic equation $x^2 - 98x + 1 = 0$. Compute the value of $\sqrt{a} + \sqrt{b}$.

2. Evaluate
$$\sum_{x=2}^{101} \sum_{y=2}^{101} \frac{1}{1 + \log_y(x) \cdot \log_{xy}(x^{\log_y(xy)})}.$$

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(f(x)) = x^2 - 2$ for all real numbers x. Given that $f(3) = 5$, compute the value of $f(7)$.

4. Let $f(x)$ be a monic polynomial of degree 12 whose roots are the integers x satisfying $-7 \le x \le -2$ or $2 \le x \le 7$. The value of
$$\left| \frac{f(i+1)}{f(i)} \right|^2$$
can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

5. The sum
$$\sum_{k=3}^{2026} \frac{1}{(k-1)\sqrt{k-2} + (k-2)\sqrt{k-1}}$$
can be expressed as a fraction $\frac{m}{n}$ in lowest terms, where m and n are relatively prime positive integers. Compute $m+n$.

6. Let x, y, and z be positive real numbers that satisfy
$$x + y + z = xyz = 2026.$$
Compute
$$\frac{(x+y)(y+z)(z+x)}{\sqrt{1+x^2}\sqrt{1+y^2}\sqrt{1+z^2}}.$$

7. Determine the sum of all possible values of $x+y$, where x and y are positive integers satisfying the equation
$$x^3 - y^3 = 13(x^2 + y^2).$$

8. Let a, b, and c be the complex roots of the cubic equation $x^3 - x^2 - x - 1 = 0$. Compute the value of
$$\left(\frac{1+a}{1-a}\right)^4 + \left(\frac{1+b}{1-b}\right)^4 + \left(\frac{1+c}{1-c}\right)^4.$$

9. For real numbers a, b, c, and d, the following equation holds:
$$\frac{3d+9}{a+b+c} = \frac{3c+16}{a+b+d} = \frac{3b-24}{a+c+d} = \frac{3a-1}{b+c+d}$$
Determine $\frac{b-c}{a-d}$, given that $a \neq d$.

10. Suppose a, b, and c are positive real numbers satisfying

$$\begin{aligned}&(12\arctan(a)+8\arctan(3a))^4+\left(4\arcsin\left(\frac{b}{\sqrt{c}}\right)\right)^4\\&+\left(\log_b\left(\left(\frac{c}{\sqrt{33}}\right)^{2\pi}\right)\right)^4+4096\left(4\arctan\left(\frac{1}{5}\right)-\arctan\left(\frac{1}{239}\right)\right)^4\\&=8\pi\,(12\arctan(a)+8\arctan(3a))\left(\log_b\left(\left(\frac{c}{\sqrt{33}}\right)^{2\pi}\right)\right)\left(4\arcsin\left(\frac{b}{\sqrt{c}}\right)\right).\end{aligned}$$

The maximum value of $(ab)^2$ can be expressed in the form $n-\sqrt{m}$, where m is a positive integer that is not necessarily squarefree. Find $n+m$.