Geometry Round Problems 2026 IMT

1. On a regular hexagon $ABCDEF$ with side length 2026^{2026}, a circle is drawn on each side such that the side is the diameter of its circle. Repeating this process for all six sides, the circles intersect at six points inside the hexagon. Connecting these points forms a new hexagon $A_1B_1C_1D_1E_1F_1$. Denote this transformation as X.
   The process X is applied repeatedly to the resulting hexagons, forming a sequence $A_iB_iC_iD_iE_iF_i$. Determine the largest number of times, a (counting the first transformation shown in the diagram), that X can be applied such that the side length of the resulting hexagon remains an integer. Compute a.

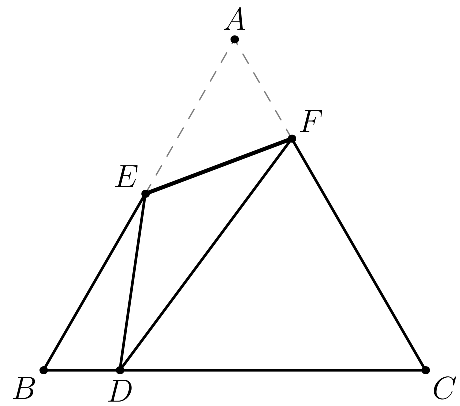
2. Let $\triangle ABC$ be an acute triangle with orthocenter H. Given that $BC = 2AH$, $AC = 3BH$, and that the area of triangle $\triangle ABC$ is 300, the value of AB can be written as $a\sqrt{b}$, where a and b are positive integers and b is square-free. Compute $a + b$.

3. Square $ABCD$ has side length 10. The center of circle Ω is located within $ABCD$ such that circle Ω is tangent to both line segment AB and diagonal BD. The largest possible radius of the circle can be expressed as $a\sqrt{b} - c$ where a, b, c are positive integers and b is squarefree. What is $a + b + c$?

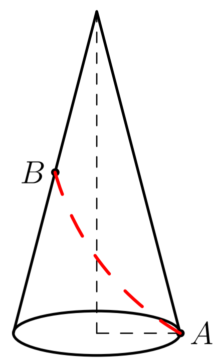
4. Triangle $\triangle ABC$ has side lengths $AB = 13$, $AC = 14$, $BC = 15$. Let M be the midpoint of side BC. Denote the cirumcircle of $\triangle ABC$ as Ω. There exists a circle ω which is internally tangent to Ω at point X and tangent to side BC at M, such that X and A lie on the same side of BC. Denote the center of ω as O. Lines AB and MX intersect at point Y. The value $\sin \angle BYO$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime. Compute $m + n$.

5. Let $ABCD$ be a convex quadrilateral with $AB \parallel CD$. Lines AD and BC are extended past A and B and intersect at point E. Given that $EA = 5$, $ED = 15$, $AB = 6$, and $EA \cdot ED = EB \cdot EC$, the circumradius of $\triangle ABC$ can be expressed as $\frac{a\sqrt{b}}{c}$, where a and c are relatively prime and b is squarefree. Compute $a + b + c$.

6. An equilateral triangle piece of paper $\triangle ABC$ with side length 15 is folded such that vertex A lands exactly on a point D on side BC. The crease line intersects AB at E and AC at F. Given that $BD = 3$, the area of $\triangle AEF$ can be expressed in the form $\frac{a\sqrt{b}}{c}$, where a and c are relatively prime positive integers and b is square-free. Compute $a + b + c$.

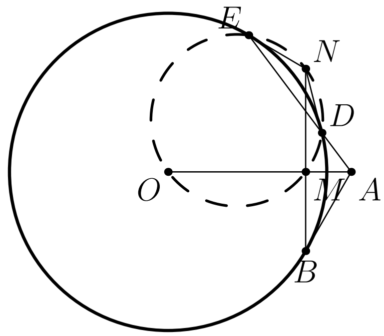


7. A solid right circular cone has a base with radius 3 and a slant height of 12. An ant starts at point A on the circumference of the base. The ant wishes to crawl along the lateral surface of the cone to reach a point B, which is located diametrically opposite of A relative to the base, and sits exactly halfway up the slant height of the cone. The square of the shortest distance the ant can travel can be expressed as $m - n\sqrt{p}$, where m, n, and p are integers and p is square-free. Find $m + n + p$.



8. In triangle $\triangle ABC$, let the side lengths be $AB = 13$, $BC = 14$, and $AC = 15$. Let I be the incenter of the triangle, and let the ray AI intersect the circumcircle of $\triangle ABC$ at a point M (where $M \neq A$). Let K be the midpoint of the segment IM. A line ℓ is drawn through K such that ℓ is perpendicular to the line AM. This line ℓ intersects the lines AB and AC at points P and Q, respectively. The area of $\triangle APQ$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime. Compute $m + n$.

9. Given a circle Γ with center O and radius 780, let A be a point such that $OA = 900$. The two tangents from A to Γ touch the circle at points B and C. A line through A intersects Γ at points D and E such that D lies between A and E, and $AD = 240$. Let M be the midpoint of BC. The circumcircle of $\triangle MDE$ intersects the line BC at M and another point N. Compute the length of MN.



10. Let $\triangle ABC$ be an acute triangle with side lengths $AB = 17$, $AC = 25$, and $BC = 28$. Let H be the orthocenter of $\triangle ABC$. Let E and F be the feet of the altitudes from B and C to the sides AC and AB, respectively. The line EF intersects the line BC at point P. The line AP intersects the circumcircle of $\triangle AEF$ at a second point X (where $X \neq A$). The value of AX^2 can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.