

1. A certain four-digit house number N is a perfect square. It has the property that if each of its digits is increased by 1, the resulting four-digit number is also a perfect square. Determine the value of N .
2. How many integers $x < 2026$ are divisible by exactly two of either 2, 3, 5, or 7?
3. Let $n = 2^{12} \cdot 5^{12}$. A positive divisor d of n is called *equidivisive* if $\tau(d) = \tau(n/d)$, where $\tau(k)$ denotes the number of positive divisors of k . Determine the number of equidivisive divisors of n .
4. Determine the number of positive integers $n \leq 2026$ such that $\lfloor \sqrt[3]{n} \rfloor$ is a divisor of n .
5. For a positive integer n , let $s_b(n)$ denote the sum of the digits of n when written in base b . Determine the number of integers n in the set $\{1, 2, 3, \dots, 1000\}$ that satisfy the equation

$$s_2(n) = s_4(2n)$$

6. The function $f(x, y)$ is defined for all integers $1 < x \leq y$. For all primes p and integers $a, b, c > 1$ the function f has the following properties:

$$f(a^b, c) = f(a, c)$$

$$f(p, p) = 1, f(a, b) = 0 \text{ if } \gcd(a, b) = 1$$

$$f(p, bc) = f(p, b) + f(p, c)$$

$$f(ab, c) = \max(f(a, c), f(b, c)) \text{ if } \gcd(a, b) = 1$$

Determine the sum of all integers $x > 1$ such that

$$f(x, 2026) \geq f(k, 2026)$$

for every integer $1 < k \leq 2026$.

7. For a positive integer n , let $\phi(n)$ denote the number of integers $1 \leq k \leq n$ such that $\gcd(k, n) = 1$. Determine the number of integers $n \in \{1, 2, \dots, 100\}$ such that $\gcd(n, \phi(n)) > 1$.
8. Positive integers a, b, c satisfy the following equation:

$$a(a^2 + 3b^2) = 4(5c - a)^3$$

How many ordered triplets (a, b, c) are there for $a \leq 2026$?

9. A token is placed at the position 0 on a number line. For a fixed positive integer n , the token performs a sequence of n jumps. In the k -th jump, for $k = 1, 2, \dots, n$, the token moves from its current position x to a new position $x + \lceil n/k \rceil$. Let L_n denote the final position of the token after all n jumps have been completed. Determine the number of positive integers $n \leq 500$ such that the final position L_n satisfies the equation:

$$L_n = \sum_{k=1}^n \lceil n/k \rceil + (n - 8)$$

10. For each positive integer n , let $f(n) = \sum_{k=1}^n \gcd(k, n)$. Determine the sum of all positive integers n such that $f(n) = 3n$.