

1. For positive integers I , M , and T satisfying $I \leq M \leq T$, the following equation holds true:

$$IMT = 5(I + M + T)$$

Compute the sum of all possible values of T across all valid (I, M, T) .

2. A polynomial of degree 6 is defined as

$$P(x) = x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

The six coefficients a_0, a_1, a_2, a_3, a_4 , and a_5 are chosen independently and uniformly at random from the set $\{1, 2, 3, 4, 5, 6\}$ by rolling six standard fair dice. Let $r_1, r_2, r_3, r_4, r_5, r_6$ be the six complex roots of $P(x)$. The expected value of the sum of the squares of the roots, $r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2$, can be expressed as a fraction $\frac{m}{n}$ in lowest terms. Compute $m + n$.

3. Brandon and Christopher play a game starting with the number 2026^{2026} on a chalkboard. On their turn, a player can subtract any proper divisor of the current number to create a new number, which replaces the old one. The player who is forced to leave the number 1 on the board loses. Assuming optimal play, the first player can always win. Compute the number of distinct integers the first player can subtract on their first turn to guarantee a win.
4. In isosceles triangle IMT with $IT = MT$ and $IM = 2026$, let the midpoint of IM be A and MT be B . The angle bisector of I bisects segment AB . The value of $\cos(T)$ can be written as $\frac{m}{n}$ for relatively prime integers m, n . Find $m + n$.
5. Compute the total number of digits of the smallest positive integer N that satisfies:
- N is a multiple of 2026.
 - N ends with the digits "2026".
 - The sum of the digits of N is 2026.
6. Three positive real numbers, x, y , and z , are chosen at random from the interval $(0, \pi)$, and satisfy the condition $x + y + z = \pi$. A triangle T is constructed with side lengths of $\sin(x)$, $\sin(y)$, and $\sin(z)$. The probability that T is strictly acute can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Determine $m + n$.
7. Determine the number of ways to fill all 25 squares of a 5×5 grid with either a 1 or a -1 such that the product of the numbers in every row is -1 , and the product of the numbers in every column is -1 .
8. Triangle ABC has $\angle B = 100^\circ$ and $AB = 1$. Let ω_1 be the circumcircle of $\triangle ABC$ with center O . A chord DE of ω_1 , distinct from AC , is drawn such that $DE \parallel AC$ and $DE = AC$. Let ω_2 be a smaller circle centered at O and tangent to both BD and DE . If F and G are the points of tangency of ω_2 with BD and DE , respectively, determine the degree measure of the central angle $\angle FOG$.
9. Let $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a continuous function such that for all positive real numbers x and y , the following equation holds:

$$f(xy) = f(x)^y \cdot f(y)^x$$

Given that $f(2) = 256$, find $\sqrt{f(3)}$.

10. The following system of equations has exactly k distinct real solution pairs (x, y) .

$$x^5 - 10x^3y^2 + 5xy^4 = \frac{117x - 44y}{x^2 + y^2}$$

$$5x^4y - 10x^2y^3 + y^5 = \frac{44x + 117y}{x^2 + y^2}$$

Let these distinct solution pairs be denoted as $(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)$. Compute

$$\sum_{j=1}^k (x_j^2 + y_j^2)^3.$$